

FUNDAMENTAL THEOREMS ON THE GENERALIZED COUPLED THERMOELASTICITY PROBLEM FOR ANISOTROPIC BODIES

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UDC 536.12:539.376

The equation of motion, Hooke's law, and the general heat-conduction equation

$$\lambda_{ij}^t t_{,ij} - c_V \dot{t} - t_0 \beta_{ij} \dot{e}_{ij} = -kw$$

for anisotropic bodies are used to derive the energy equation

$$\frac{d}{dt} (K + W + P) + X_t = \int_{\Omega} v_i X_i dV + \int_S p_i v_i dS + \frac{1}{t_0} \int_{\Omega} t k w dV + \frac{1}{t_0} \int_S \lambda_{ij}^t t_{,j} n_i dS, \quad (1)$$

which expresses the generalized energy conservation principle.

Here λ_{ij}^t are the thermal conductivity coefficients; t is the temperature; c_V is the specific heat at constant volume; $l = 1 + \tau_R (\partial / \partial \tau)$; τ_R is the heat flux relaxation time; τ is the time; t_0 is the temperature of the body in the unstrained state; $\beta_{ij} = c_{ijkl} \alpha_{kl}^t$; α_{kl}^t are the coefficients of thermal expansion; \dot{e}_{ij} is the strain rate; w is the density of internal heat sources; X_i are the inertial forces; $v_i = \dot{u}_i$; u_i are the components of the displacement vector; p_i are the applied pressures on the surface S of the body Ω ; $K = \rho / 2 \int_{\Omega} v_i v_i dV$ is the kinetic energy; $W = \frac{1}{2} \int_{\Omega} c_{ijkl} e_{kl} e_{ij} dV$ is the work of deformation; $P = c_V / 2 t_0 \int_{\Omega} t^2 dV$ is the thermal energy function; $X_t = \frac{1}{t_0} \int_{\Omega} \lambda_{ij}^t t_{,j} t_{,i} dV + \tau_R \int_{\Omega} \dot{s} t dV$ is the generalized dissipation function; s is the entropy; and ρ is the density.

The generalized conservation principle (1) differs from the classical dissipation function by a term containing the acceleration of entropy growth of the system.

The generalized energy theorem is used to prove the uniqueness of solution of the fundamental equations for the generalized dynamic coupled thermoelasticity of anisotropic bodies in the presence of heat sources and inertial forces.

The variation of the work of deformation, Hooke's law, the generalized Fourier equation, and the entropy growth-rate relation for anisotropic bodies are used to prove a variational theorem on the generalized coupled thermoelasticity of anisotropic bodies; this theorem states that the variation of the sum of the work of deformation, heat flux, and generalized dissipation function is equal to the virtual work of the external forces, inertial forces, and heating of the surface of the body.

A reciprocity theorem is proved for the generalized coupled thermoelasticity of anisotropic bodies.

Dep. 3413-75, September 29, 1975.

Original article submitted November 10, (1972).

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APPROXIMATE ANALYTICAL SOLUTION OF THE
NONSTEADY HEAT-CONDUCTION PROBLEM IN A
VARIABLE-TEMPERATURE MEDIUM

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UDC 536.2.023

For the determination of the temperature field $t(r, z, \tau)$ of a finite cylinder in a variable-temperature medium the duration of the investigated thermal process is partitioned into a finite number of j -th intervals of arbitrary duration $\Delta\tau_j$, in which the temperature of the medium is approximated by a linear law.

A solution of the heat-conduction equation for the finite cylinder $0 < r \leq R$, $-h \leq z \leq h$

$$\frac{\partial t(r, z, \tau)}{\partial \tau} = a \left[\frac{\partial t(r, z, \tau)}{r \partial r} - \frac{\partial^2 t(r, z, \tau)}{\partial r^2} + \frac{\partial^2 t(r, z, \tau)}{\partial z^2} \right] \quad (1)$$

subject to the boundary conditions

$$t(r, z, 0) = t_0, \quad (2)$$

$$t(R, z, \tau) = t(r, \pm h, \tau) = t_{av}, \quad (3)$$

in which t_{av} is the time function $t_{av} = \varphi(\tau)$, is found by applying the Duhamel integral theorem to the solution for a finite cylinder under constant-temperature conditions $U(r, z, t)$ [1]:

$$t(r, z, \tau) - t_0 = (t_{av}^{\tau} - t_0) U(r, z, 0) + \int_0^{\tau} (t_{av}^{\eta} - t_0) U'(r, z, \tau - \eta) d\eta, \quad (4)$$

where τ is the duration of the investigated thermal process.

When the boundary condition is approximated by a piecewise-linear time function, the expression for the function $\varphi(\tau)$ in the i -th interval $\Delta\tau_i$ is written in the form

$$\varphi(\tau) - t_0 = (\varphi_{i-1} - t_0) + b_i (\tau - \tau_{i-1}), \quad (5)$$

$$b_i = (\varphi_i - \varphi_{i-1}) / \Delta\tau_i. \quad (6)$$

The computation of the integral in (4) is carried out by successive integration over the time intervals in which the function t_{av} is considered to be linear. Combining the integration results for all intervals and performing a series of appropriate transformations, we obtain

$$\frac{t(r, z, \tau) - t_0}{t_0} = \frac{t_{av} - t_0}{t_0} \Phi(Fo_h^{\tau}) + \sum_{i=1}^j Pd_i [Fo_h^i \Omega(Fo_h^i) - Fo_h^{i+1} \Omega(Fo_h^{i+1})], \quad (7)$$

where $Pd_i = b_i h^2 / at_0$ is the Predvoditelev number and $Fo_h^i = a(\tau - \tau_{i-1}) / h^2$ is the Fourier number.

When the function t_{av} is approximated by a step function, the required solution takes the form

$$\frac{t(r, z, \tau) - t_0}{t_0} = \sum_{i=1}^j d_i \Phi(Fo_h^i), \quad (8)$$

where

$$d_i = (\varphi_i - \varphi_{i-1}) / t_0. \quad (9)$$

To facilitate the practical application of the solutions obtained here a numerical calculation of the dimensionless functions $\Phi(Fo_h)$ and $\Omega(Fo_h)$ is carried out. The computer-calculated values of the functions for the center of the finite cylinder are given in tables for Fourier numbers ranging from 0.0100 to 100 and for cylinders with geometric dimensions $k = h/R$ ranging from 0.1875 to 2.

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Dep. 148-76, December 1, 1975.

Original article submitted June 24, 1975.

THERMAL FIELD OF A SPHERICAL SOURCE IN
A HETEROGENEOUS CONTINUUM

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UDC 536.12

The propagation of heat by molecular heat conduction in a two-component medium is investigated. The components of the medium have different thermophysical characteristics, and heat transfer takes place between them. The heat-transfer process between the components is assumed to be quasisteady, so that the transfer rate is proportional to the temperature difference between the components.

The problem of the temperature field outside a sphere having a certain radius with a given heat flux to it is solved. Expressions are derived by an operational method for the temperatures of both components when the latter have arbitrary parameters and the radius r_0 of the spherical source is small.

Then the case of a medium in which $m_1 k_1 \gg m_2 k_2$, where m_1 and m_2 are the volume or surface fractions occupied by the respective components and k_1 and k_2 are the thermal conductivities, is discussed for arbitrary r_0 . This case is important in connection with the obvious fact that the temperature difference of the components increases with the difference in their thermal conductivities.

Next, the limiting case in which $m_1 k_1 \gg m_2 k_2$ but $m_1 \rho_1 c_1 \ll m_2 \rho_2 c_2$ is analyzed, where ρ_1 and ρ_2 are the densities and c_1 and c_2 are the specific heats of the respective components. Inasmuch as ρc varies only slightly for the majority of solids and liquids, the second stated condition is satisfied when $m_1 \ll m_2$, i.e., when the portion of the medium occupied by the first component with the higher thermal conductivity is small.

Finally, an expression is obtained for the total net heat flux between the components per unit time throughout the entire volume:

$$W(t) = 4\pi\alpha \int_{r_0}^{\infty} [T_2(r, t) - T_1(r, t)] r^2 dr,$$

where T_1 and T_2 are the temperatures of the components, r is the space coordinate, t is the time, and α is the heat-transfer coefficient between the components. For the case of arbitrary parameters of the medium $W(t)$ is expressed in the form

$$W(t) = -\frac{\alpha(1-\varepsilon\delta)}{m_1\rho_1c_1(1+\varepsilon)} \int_0^t f(\theta) \exp\left[-\frac{\alpha(1+\delta)(t-\theta)}{m_1\rho_1c_1}\right] d\theta,$$

where $\varepsilon = m_2 k_2 / m_1 k_1$; $\delta = m_1 \rho_1 c_1 / m_2 \rho_2 c_2$; and $f(t)$ is the source strength.

Curves of $T_1(r, t)$, $T_2(r, t)$, and $W(t)$ are plotted for selected values of the parameters.

It is important to note that allowance for the difference in the temperatures T_1 and T_2 of the components is essential for strongly nonsteady heat-transport processes for process times $t \lesssim \tau_1$, where $\tau_1 = m_2 \rho_2 c_2 (1 + \varepsilon \delta) [\alpha (1 + \varepsilon)]^{-1}$ is the delay time constant for temperature redistribution processes in the heterogeneous continuum.

Dep. 3412-75, July 22, 1975.

Original article submitted June 6, 1974.

INFLUENCE OF DEPARTURE FROM ONE-DIMENSIONAL
CONDITIONS IN DETERMINING THE THERMAL CONDUCTIVITY
OF LIQUIDS BY THE METHOD OF COAXIAL CYLINDERS

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UDC 536.2.08

The method most often used in experimental studies of the thermal conductivity of liquids at high temperatures is the steady-state method of coaxial cylinders [1]. This paper discusses the influence of the inherent departure from one-dimensionality in experimental realization of the method under conditions of non-isothermicity of the lateral surfaces of the cylinders.

Our analysis is based on the solution of the problem of the steady-state temperature field of a three-layer cylinder of finite length with an arbitrary distribution of the temperature lengthwise along the outer surface and radially at the end surface as well as an arbitrary lengthwise distribution of the heat flux on the inner surface of the cylinders.

The sought-after function must vanish at the ends of the cylinders. Then the method of separation of variables (Fourier method) is used to find the radius-dependent coefficients of the function from the inhomogeneous modified Bessel equation of order zero [2], which becomes homogeneous when the temperature at the ends is specified as a logarithmic function. The coefficients of the solution of the Bessel equation are found from the boundary conditions and the contact conditions at the lateral surfaces with those conditions expanded into a Fourier sine series.

The calculations are carried out on a BÉSM-4M computer. The temperature difference in the layer and, with the aid of the formula for the method of coaxial cylinders, the thermal conductivity of the liquid are computed for given geometric dimensions, thermal conductivities of the cylinders (including the liquid layer), and boundary conditions. The error incurred in the results of the measurements due to inhomogeneity is determined by comparing the computed and specified values of the thermal conductivity.

The influence of such factors as nonisothermicity of the lateral surface, the inequality of thermal insulation of the ends, the value of the heat flux at the inner surface, and the lengths, thicknesses, and thermal conductivities of the cylinder materials on the error is investigated.

Nonisothermicity significantly increases the error of determination of the thermal conductivity. Axial leakages of heat from the central zone in which the measurements are performed are possible not only through the ends, but also through the peripheral parts of the lateral surface of the cylinders.

The presence of the inner cylinder between the axial heater and the liquid strongly affects the error. The latter increases with the thicknesses and thermal conductivities of the cylinders.

For a nonisothermicity up to 2°C, which is deemed acceptable by many researchers, the error attains 2 or 3% for a length-to-diameter ratio of 8 to 10 and can attain 8 to 10% when that ratio is diminished to 5.

The results of the study make it possible to estimate the error due to nonisothermicity for specific experimental conditions attending the determination of the thermal conductivity of liquids.

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Dep. 299-76, December 29, 1975.

Original article submitted November 19, 1975.

The thermal conductivity of melts of aluminum electrolyzer electrolyte and its main constituent, cryolite, has previously been investigated by the steady-state method of coaxial cylinders [1]. To assess the influence of radiative heat transfer the effective thermal conductivity of cryolite is determined for three thicknesses of the melt layer (mm): 1.53, 2.06, 2.67 in the temperature range from 1000 to 1100°C, and the same is determined for the electrolyte in layers with thicknesses of 1.57 and 2.78 mm in the temperature range from 940 to 1040°C. The material of the cylinders is GMZ graphite, whose surface is packed with pyrographite to increase its resistance to the aggressive melts. The maximum analytical error of determination of the thermal conductivity is mainly attributable to the error of determination of the temperature difference in the layer and to end losses and is equal to 10%.

The experimental results indicate an increase in the effective thermal conductivity with the layer thickness, an effect that can be explained by radiative heat transfer. The true (conductive) thermal conductivity and absorption coefficient of the melts are calculated in the gray-body approximation according to the equation of Poltz [2] on the basis of the dependence of the effective thermal conductivity on the layer thickness. The emissivity of the graphite and the refractive index of the melts are evaluated in accordance with [3, 4]. In the investigated temperature range the true thermal conductivity of cryolite increases from 0.75 to 0.85 W/m·°K, while that of the electrolyte increases from 0.65 to 0.70 W/m·°K. The absorption coefficient lies in the interval 0.6 to 0.4 mm⁻¹ for cryolite and is about 0.6 mm⁻¹ for the electrolyte.

The spectral absorption is also determined experimentally by the double specimen method [5] in the interval of wavelengths from 1 to 6 μ; for this operation we have developed a windowless cell, in which a flat melt layer is maintained by surface tension, along with a high-temperature oven attachment to fit the IKS-21 infrared spectrometer. It is found that the absorption coefficient can be assumed constant in the investigated wavelength interval. The greater absorption of the molten industrial electrolyte is attributable to its inferior purity and the presence of aluminum oxide in it.

The use of the resulting values of 0.3 mm⁻¹ for cryolite and 0.4 mm⁻¹ for the electrolyte in calculating the true thermal conductivity according to Poltz's equation yields results that are about 10 to 15% lower than the cited values. This discrepancy can be explained by the approximative character of the analytical method, errors in the data taken from the literature, and the lower transmissivity of the melt in the thermal conductivity determinations as a result of graphite particle inclusions and other impurities.

The results of the study show that for the investigation of the thermal conductivity and heat transfer in cryolite melts it is necessary to take into account the influence of radiation. The effective thermal conductivity measured for cryolite melts in a graphite cell by the nonsteady-state method of coaxial cylinders turns out to be more than double the true thermal conductivity.

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Dep. 298-76, December 29, 1975.

Original article submitted November 19, 1975.

POWER EXPONENT FOR THE PRANDTL NUMBER UNDER FREE-CONVECTION MELTING CONDITIONS

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UDC 536.421.1:536.252

Results are given from an experimental determination of the power exponent of Pr in the melting of ice, benzene, hexadecane, pentadecane, and cyclohexane in a natural melt. The experiments are carried out for two values of the Grashof number: $Gr = 3 \cdot 10^6$ and $1.88 \cdot 10^5$, permitting maximum utilization of the established data of other authors.

Inasmuch as the boundary conditions imply that the heat transfer of a porous body is proportional to the first power of ρ/ρ_s , the experimental results are processed to give $Nu/(\rho/\rho_s)$ as a function of Pr for $Gr = \text{const}$. The power of the Prandtl number turns out to be close to 2, indicating the dominant influence of inertial forces under free-convection melting conditions. This effect is attributed to gas liberation, which promotes melting, because the frozen medium always contains gas bubbles.

An increase in the power exponent of the Prandtl number due to artificially generated turbulence has been noted earlier in the examples of mass transfer associated with rectification and absorption [1] and heat transfer in the presence of wire turbulence promoters in a single-phase medium with forced convection [2].

NOTATION

Nu , Nusselt number; Gr , Grashof number; Pr , Prandtl number; ρ , density of the melt at the average boundary-layer temperature, kg/m^3 ; ρ_s , bulk density of the solid phase, kg/m^3 .

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Dep. 149-76, November 19, 1975.

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Original article submitted February 19, 1975.

NUMERICAL SOLUTION OF THE PROBLEM OF TWISTED GAS FLOW IN A PIPE WITH INJECTION IN THE INITIAL SECTION

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UDC 533.697.001.57:681.322.2

A finite-difference method is proposed for solving the equations of motion of twisted flow in a pipe in whose initial section a secondary gas flow is injected at a higher temperature. Secondary flow injection is realized by means of a swirling device in the form of a volute chamber with cascaded guide vanes.

The numerical solution is based on the substitution of average variables for the values of the axial and radial flow velocity components in the convective terms of the Navier-Stokes equations [1] with reliance on the "effective viscosity" hypothesis [2].

The equations are solved in matrix form, and the matrix of coefficients has a relatively simple (three-diagonal) form. This feature is an asset of the proposed numerical technique over the one proposed in [3].

A special program for the M-222 digital computer has been written to implement the calculations by the proposed method. The influence of flow twisting (swirling) on the mixing of gases in the swirler zone is investigated by computer modeling.

It is found that the main factor affecting the distributions of the temperature and velocity fields is the tangential component of the secondary-flow injection rate at the exit from the guide-vane cascade. The value of that velocity (β) is determined by the mass flow of gas through the swirler and the geometrical parameters

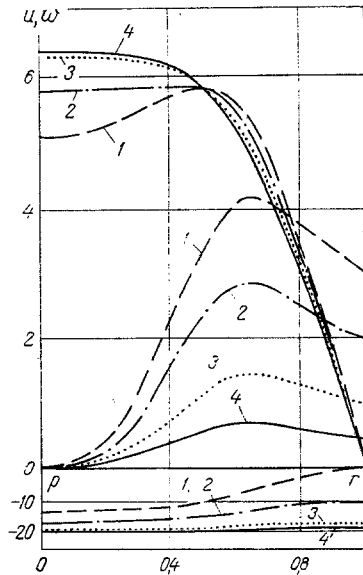


Fig. 1. Influence of an increase in the tangential injection velocity on the distributions of the static pressure and axial and tangential velocities for $\Psi = 0.75$ at the swirler exit. 1) $\beta = 3$; 2) 2; 3) 1; 4) 0.5.

of the cascade. Figure 1 shows the distributions of the axial and tangential flow velocities (u, w) and the static pressure (p) when the ratio of the mass flow of gas through the swirler to the total mass flow through the system is $\Psi = 0.75$. It is seen that an increase in β produces a dip in the axial-flow velocity profile. It is possible for reverse currents to form in the event of large swirling. An increase in the swirling rate intensifies longitudinal mixing and heat transfer between the secondary and primary gas flows. It is noted that an increase in the ratio of the radial to the tangential component of the injection velocity shifts the tangential maximum toward the pipe axis, inhibiting the inception of reverse currents of the gas.

A comparison of the modeling results with experimental data shows that the proposed finite-difference method affords a satisfactory description of twisted flow in an initial section of the order of six pipe radii.

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Dep. 1218-75, March 31, 1975.

Original article submitted October 22, 1974.

ADVANCEMENT OF THE MOISTURE CHANGE-OF-STATE FRONT IN THE FREEZING AND THAWING OF SOILS

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UDC 624.138.35+624.139.64

The solution of problems such as that named in the title is extremely relevant to the artificial thawing of soils and construction in permafrost regions. Approximate solutions are investigated for the radial freezing of a soil around a single icing column for $t_u > t_F$, with series expansion of the solutions, and one-dimensional soil thawing problems are analyzed for the case $t_f < t_F$.

The proposed solution for the radial freezing process has the distinctive feature that the integral characterizing the time variation of the gradient for the unfrozen zone

$$\frac{\partial t_u}{\partial r} = \frac{(t_u - t_F)}{\rho} \frac{2}{\pi} \int_0^{\infty} \exp\left(\frac{a\tau}{\rho^2}\right) \frac{J_1(v) Y_0(v) - J_0(v) Y_1(v)}{J_0^2(v) + Y_0^2(v)} dv \quad (1)$$

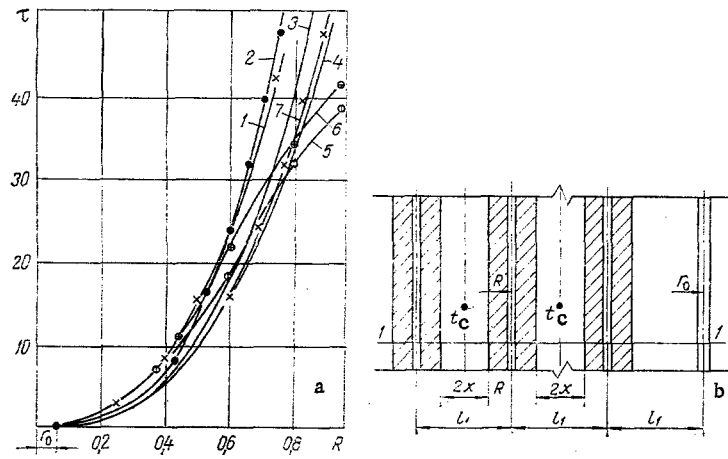


Fig. 1. a) Duration of freezing of the ice-soil zone. 1) According to Eqs. (3) and (4); 2) according to Charnyi; 3) according to Khakimov for $a = 4$; 4) the same for $a = 5$; 5) according to Trupak's equation when $t_c = 0$ for $R_0 = 0.375l$; 6) the same when $t_c = 0$ for $R_0 = 0.45l$; 7) solution of the problem of V. S. Luk'yanov's "hydrintegrator." b) Section along the frost curtain (the frozen zone is hatched, τ is in days, and R in meters).

is approximated in the interval $Fo = 0.5$ to 25 by the expression

$$\frac{2}{\pi} \int_0^{\infty} f(Fo, v) \approx A_r \frac{B_r}{\sqrt{\frac{a\tau}{\rho^2}}} \approx 0.37 + \frac{0.61}{\sqrt{\frac{a\tau}{\rho^2}}}, \quad (2)$$

whereupon the known boundary condition at the freezing front [1] is used and the expression integrated to obtain

$$\tau = \frac{l^2}{a_u} z, \quad (3)$$

$$z = \left[\frac{\lambda_u Y B_r k}{\lambda_f - \lambda_u Y A_r k} - \sqrt{\frac{q_0 a_u k}{(t_F - t)(\lambda_f - \lambda_u Y A_r k)}} \right]^2, \quad (4)$$

$$q_0 = [q(W - w_u) \gamma_f + 0.5 C_f (t_F - t) + C_u (t_u - t_F)]. \quad (5)$$

Figure 1a compares the result of calculations based on Eqs. (3)-(5) with the well-known solutions Charnyi [2] and Khakimov [3]. The result is close to the former. In the case of a row of icing columns walls of unfrozen soil with a thickness $2x$ remain between the generatrices of the cylindrical ice-soil interfaces (Fig. 1b). In the case of an infinite wall at which $t_w = 0$ for $Fo > 2$ the temperature at its center $t_c \rightarrow 0$. Consequently, after $R \geq 0.25l$ it is permissible to carry out the calculations according to equations based on the assumption that $t_u = 0$ [2, 4].

For one-dimensional thawing of frozen ground solutions are obtained for boundary conditions of the first and third kind. In this case $A_0 = 0$ and $B_0 = 0.56$. The solutions are identical to (3) and (4). For the boundary conditions of the first kind z is determined from the equation

$$z - \varphi B_0 Y \psi \sqrt{z} - 0.5 Ko = 0. \quad (6)$$

NOTATION

J, Y , Bessel functions; τ , time; a_f, a_u , thermal diffusivities of the frozen and unfrozen soils; t_u, t_f , in situ temperatures of the freezing (unfrozen) and thawing (frozen) soils; t_F , soil freezing point; t, t_w, t_c , temperatures of icer, wall surface, and center of the wall; $Y = (t_u - t_f) / (t_F - t)$; ρ , time-varying radius; R , freezing radius; r_0 , radius of icer; $k = (R - r_0) / R$; q , heat of crystallization of water; W , soil moisture content by weight; w_u , unfrozen-water content of soil; γ_f , bulk density of soil matrix; C_f, C_u , heat capacities at constant volume of the frozen and unfrozen soils; $\varphi = C_f / C_u$; $\psi = \sqrt{a_f / a_u}$; $\lambda_u = a_u C_u$; $\lambda_f = a_f C_f$; Fo , Fourier number for the unfrozen zone; Ko , Kossovich number; in Eq. (3) $l = R$ for the radial problem, $l = H$ for the one-dimensional problem; q_0 , variation of soil heat content at freezing front.

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Dep. 330-76, July 24, 1975.

Original article submitted May 29, 1975.

EMISSIVITY OF GASES: SULFUR DIOXIDE AND CARBON MONOXIDE

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UDC 536.3:535.343

Equations are substantiated for the emissivity of two gases: sulfur dioxide and carbon monoxide. The emphasis is on simplification of the equations. All calculations are carried out on a Promin' digital computer. The basic input comprises experimental data on three parameters of the spectral bands. Consequently, the nonisothermicity and nonisobaricity of the gas volumes can be taken into account by means of the three-parameter approximation.

The results are independent for sulfur dioxide. They do not have the corrections according to the data of earlier researchers: Hottel; Chen and Tien; Balakrishnan and Edwards; A. N. Golitsyn. A critical examination of works published to date shows them to be insufficient and in some cases inconsistent. The present method and results differ significantly from those published earlier. By contrast with the analysis of Chen and Tien: 1) original expressions are used for the integral absorption in separate spectral bands; 2) a statistical model of the spectrum is adopted; 3) the increase in the number of strong lines due to population growth of the upper vibrational levels is taken into account; 4) the rotational structure parameter of the band ω_3 is considerably decreased to conform more realistically with the influence of pressure on the absorption; 5) the effective pressure equation is changed; 6) the integral intensity of the band $\omega_1 + \omega_3$ is increased from 9 to 33 $\text{cm}^{-1} \cdot \text{cm} \cdot \text{bar}$.

The emission spectrum of carbon monoxide is simple and has been thoroughly studied. This gas is the object of tests and comparisons of analytical methods. In the present calculation theoretical innovations are also tested: 1) a generalized statistical narrowband model is developed and applied; 2) the regular and statistical narrowband models have become better reconciled with increasing temperature; 3) the shifts of the subbands of upper vibrational levels are taken into account indirectly. The integral absorption in the fundamental spectral band is calculated from the experimental data of Burch and Williams, Abu-Romia, and Tien. The present calculation is simpler than calculations of the Greif group and exhibit better agreement with the experimental data.

The main advances of applied spectroscopy that are applicable to the problem are used in the study. Consequently, the risk of extrapolating the experimental data on the temperature and optical thickness is minimized. It nonetheless remains large. The existing experimental base is too limited to provide more reliable results, particularly with regard to sulfur dioxide.

Dep. 297-76, August 5, 1975.

Original article submitted May 8, 1975.